(b) $$\int \frac{dx}{\sqrt{x(1-2x)}} = \int \frac{dx}{\sqrt{x-2x^2}} = \frac{1}{\sqrt{2}}\int \frac{dx}{\sqrt{\frac{x}{2}-x^2}}$$

$$= \frac{1}{\sqrt{2}}\int \frac{dx}{\sqrt{\frac{1}{16}-\left[x^2-\frac{x}{2}+\frac{1}{16}\right]}}$$

$$= \frac{1}{\sqrt{2}}\int \frac{dx}{\sqrt{\left(\frac{1}{4}\right)^2-\left(x-\frac{1}{4}\right)^2}} = \frac{1}{\sqrt{2}}\sin^{-1}\left\{\frac{\left(x-\frac{1}{4}\right)}{\left(\frac{1}{4}\right)}\right\}+c$$

$$= \frac{1}{\sqrt{2}}\sin^{-1}(4x-1)+C$$

ଆସ ନିଜେ ନିଜକୁ ପରଖିବା 26.5

1. ମାନ ନିର୍ଣ୍ଣୟ କର :

(a) $\int \frac{x^2}{x^2-9}dx$ (b) $\int \frac{e^x}{e^{2x}+1}dx$ (c) $\int \frac{x}{1+x^4}dx$

(d) $\int \frac{dx}{\sqrt{16-9x^2}}$ (e) $\int \frac{dx}{1+3\sin^2 x}$ (f) $\int \frac{dx}{\sqrt{3-2x-x^2}}$

(g) $\int \frac{dx}{3x^2+6x+21}$ (h) $\int \frac{dx}{\sqrt{5-4x-x^2}}$ (i) $\int \frac{dx}{x\sqrt{3x^2-12}}$

(j) $\int \frac{d\theta}{\sin^4\theta+\cos^4\theta}$ (k) $\int \frac{e^x\,dx}{\sqrt{1+e^{2x}}}$ (*l*) $\int \sqrt{\frac{1+x}{1-x}}\,dx$

(m) $\int \frac{dx}{\sqrt{2ax-x^2}}$ (n) $\int \frac{3x^2}{\sqrt{9-16x^6}}dx$ (o) $\int \frac{(x+1)}{\sqrt{x^2+1}}dx$

(p) $\int \frac{dx}{\sqrt{9+4x^2}}$ (q) $\int \frac{\sin\theta}{\sqrt{4\cos^2\theta-1}}d\theta$ (r) $\int \frac{\sec^2 x}{\sqrt{\tan^2 x-4}}dx$

(s) $\int \frac{1}{(x+2)^2+1}dx$ (t) $\int \frac{1}{\sqrt{16x^2+25}}dx$

26.6 ଭାଗ ଭାଗ କରି ସମାକଳନ (Integration by parts)

ଅବକଳନ ପଢ଼ିବା ବେଳେ ତୁମେ ଶିଖିଛ ଯେ

$$\frac{d}{dx}(fg) = f\frac{d}{dx}(g) + g\frac{d}{dx}(f)$$

ବା, $f\frac{d}{dx}(g) = \frac{d}{dx}(fg) - g\frac{d}{dx}(f)$ ... (1)

ତୁମେ ଆହୁରି ମଧ୍ୟ ଜାଣିଛ ଯେ $\int \frac{d}{dx}(fg)\,dx = fg$

(1) ର ସମାକଳନ କରିବା ଦ୍ୱାରା ଆମେ ପାଇବା

$$\int f\frac{d}{dx}(g)\,dx = \int \frac{d}{dx}(fg)\,dx - \int g\frac{d}{dx}(f)\,dx$$

$$= fg - \int g\frac{d}{dx}(f)\,dx \quad \text{... (2)}$$

ଯଦି ଆମେ ନେଉ $f = u(x); \frac{d}{dx}(g) = v(x)$

(2) ହେବ $\int u(x)\,v(x)\,dx = u(x)\cdot\int v(x)\,dx - \int\left[\frac{d}{dx}(u(x))\int v(x)\,dx\right]dx$

= ପ୍ରଥମ ଫଳନ × ଦ୍ୱିତୀୟ ଫଳନର ସମାକଳ - ∫ [ପ୍ରଥମ ଫଳନର ଅବକଳ

× ଦ୍ୱିତୀୟ ଫଳନର ସମାକଳ]dx

A B

ଏଠାରେ ଗୁରୁତ୍ୱପୂର୍ଣ୍ଣ କାର୍ଯ୍ୟ ହେଉଛି ଫଳନ ଦୁଇଟିର ଗୁଣନରୁ ପ୍ରଥମ ଓ ଦ୍ୱିତୀୟ ଫଳନ ବାଛିବା, କାରଣ ଦୁଇଟି ମଧ୍ୟରୁ ଯେକୌଣସି ଫଳନକୁ ପ୍ରଥମ ଓ ଅନ୍ୟଟିକୁ ଦ୍ୱିତୀୟ ରୂପେ ବଛା ଯାଇପାରେ ।

ଏହି ବାଛିବା ପ୍ରକ୍ରିୟା ଲାଗି ଉପରେ ଦର୍ଶାଯାଇଥିବା B ଚିହ୍ନିତ ଅଂଶ ହେବ ସୂଚକ । ପ୍ରଥମ ଫଳନ ଏପରି ବଛାଯିବା ଉଚିତ୍ ଯେପରି ଅବକଳନ ପରେ ଏହା ଏକ ନିମ୍ନତର ପଦ ବା ଧ୍ରୁବକ ପଦରେ ପରିଣତ ହେବ ।

$x \sin x$, $x \cos^2 x$, $x^2 e^x$ ଭଳି ଫଳନର ସମାକଳନରେ

(i) ବୀଜଗଣିତୀୟ ଫଳନକୁ ପ୍ରଥମ ଫଳନ ରୂପେ ନେବା ଆବଶ୍ୟକ ।

(ii) ଯଦି ବୀଜଗଣିତୀୟ ଫଳନ ନ ଥାଏ, ତେବେ ଏକ ଫଳନ ଖୋଜ ଯାହାକି ପୂର୍ବୋକ୍ତ 'B'ରେ ଥିବା ଗୁଣଫଳକୁ ସରଳ କରିଦେବ; ପ୍ରଥମ ଫଳନ ବାଛିବା କାର୍ଯ୍ୟକୁ ନିମ୍ନ କ୍ରମରେ କରାଯାଇପାରେ :

(i) ଏକ ପ୍ରତିଲୋମ ଫଳନ

(ii) ଏକ ଲଘୁଗଣକୀୟ (ଲଗାରିଦ୍ମୀୟ) ଫଳନ

(iii) ଏକ ତ୍ରିକୋଣମିତୀୟ ଫଳନ (iv) ଏକ ଘାତାଙ୍କୀୟ ଫଳନ

ନିମ୍ନ ଉଦାହରଣମାନଙ୍କ ମାଧ୍ୟମରେ ପ୍ରଥମ ଫଳନ ବାଛିବା ଲାଗି ଉପଯୁକ୍ତ ଧାରଣା ନିମିତ୍ତ ଅଭ୍ୟାସ କରିବାର ସୁଯୋଗ ମିଳିବ ।

		ପ୍ରଥମ ଫଳନ	ଦ୍ୱିତୀୟ ଫଳନ
1.	$\int x\cos x\,dx$	x (ବୀଜଗଣିତୀୟ ହେତୁ)	$\cos x$
2.	$\int x^2 e^x\,dx$	x^2 (ବୀଜଗଣିତୀୟ ହେତୁ)	e^x
3.	$\int x^2 \log x\,dx$	$\log x$	x^2
4.	$\int \frac{\log x}{(1+x^2)}\,dx$	$\log x$	$\frac{1}{(1+x)^2}$
5.	$\int x\sin^{-1}x\,dx$	$\sin^{-1}x$	x
6.	$\int \log x\,dx$	$\log x$	1 (ଗୋଟିଏ ମାତ୍ର ଲଘୁଗଣକୀୟ ଫଳନ ଓ ପ୍ରତିଲୋମ ତ୍ରିକୋଣମିତୀୟ ଫଳନ କ୍ଷେତ୍ରରେ 'ଏକ'କୁ ଆମେ ଦ୍ୱିତୀୟ ଫଳନ ରୂପେ ନେଇଥାଉ)
7.	$\int \sin^{-1}x\,dx$	$\sin^{-1}x$	1

ଉଦାହରଣ 26.37

ମାନ ନିର୍ଣ୍ଣୟ କର : $\int x\cos x\,dx$

ସମାଧାନ : ପଲିନୋମିଆଲ (ବୀଜଗଣିତୀୟ ଫଳନ) xକୁ ପ୍ରଥମ ଫଳନ ଏବଂ ତ୍ରିକୋଣମିତିକ ଫଳନ cos x କୁ ଦ୍ୱିତୀୟ ଫଳନ ରୂପେ ନେଇ, ଆମେ ପାଉ :

$$\int x\cos x\,dx = x\int \cos x\,dx - \int\left[\frac{d}{dx}(x)\cdot\int \cos x\,dx\right]dx$$

$$(\text{୧ମ ଫଳନ f ୨ୟ ଫଳନର ସମାକଳ} - \int\left[\frac{d}{dx}(\text{୧ମ ଫଳନ})\cdot\int(\text{୨ୟ ଫଳନ}\ dx)\right]$$

$$= x\sin x - \int 1\cdot\sin x\,dx$$

$$= x\sin x - [-\cos x] + C$$

$$= x\sin x + \cos x + C$$

ଉଦାହରଣ 26.38

ମାନ ନିର୍ଣ୍ଣୟ କର : $\int x^2 \sin x\,dx$

ସମାଧାନ : ବୀଜଗଣିତୀୟ ଫଳନ $\sin x$ କୁ ୧ମ ଫଳନ $= -x^2 \cos x + 2\int x\cos x\ dx$ ଏବଂ x^2 କୁ ୨ୟ ଫଳନ ରୂପେ ନେଇ, ଆମେ ପାଉ :

$$\int \underset{\text{୨ୟ}}{x^2} \underset{\text{୧ମ}}{\sin x}\, dx = x^2 \int \sin x - \int\left[\frac{d}{dx}(x^2)\int \sin x\, dx\right]dx$$

$$= -x^2 \cos x - 2\int x\cos x\, dx$$

$$= -x^2 \cos x + 2\int x\cos x\ dx \qquad ....(1)$$

ପୁନଶ୍ଚ, $\int x\cos x\, dx = x\sin x + \cos x + C ... (2)$

(2) ରେ (1)କୁ ପ୍ରତିସ୍ଥାପନ କରି ଆମେ ପାଇବା

$$\int x^2 \sin x\, dx = -x^2 \cos x + 2[x\sin x + \cos x] + c$$

$$= -x^2 \cos x + 2x \sin x + 2 \cos x + C$$

ଉଦାହରଣ 26.39 ମାନ ନିର୍ଣ୍ଣୟ କର : $\int x^2 \log x\, dx$

ସମାଧାନ : ଅଗ୍ରାଧିକାର ନିୟମ ଅନୁଯାୟୀ log xକୁ ୧ମ ଫଳନ ରୂପେ ନିଆଯିବ ।

$$\therefore \int \underset{\text{୧ମ}}{\log x} \cdot \underset{\text{୨ୟ}}{x^2}\, dx = \frac{x^3}{3}\log x - \int \frac{1}{x}\cdot\frac{x^3}{3}dx$$

$$= \frac{x^3}{3}\log x - \int \frac{x^2}{3}dx$$

$$= \frac{x^3}{3}\log x - \frac{1}{3}\left(\frac{x^3}{3}\right) + C$$

$$= \frac{x^3}{3}\log x - \frac{x^2}{9} + C$$

ଉଦାହରଣ 26.40 ମାନ ନିର୍ଣ୍ଣୟ କର : $\int \frac{\log x}{(1+x)^2}\ dx$

ସମାଧାନ : $\int \frac{\log x}{(1+x)^2}dx = \int \underset{\text{୧ମ}}{\log x} \cdot \underset{\text{୨ୟ}}{\frac{1}{(1+x)^2}}dx$

$$= \log x\left(-\frac{1}{1+x}\right) - \int \frac{1}{x}\times\left(\frac{-1}{1+x}\right)dx$$

$$= \frac{-\log x}{1+x} + \int \frac{1}{x(1+x)}dx$$

$$= \frac{-\log x}{1+x} + \int\left[\frac{1}{x} - \frac{1}{1+x}\right]dx$$

$$= \frac{-\log x}{1+x} + \int \frac{1}{x} dx - \int \frac{1}{1+x} dx$$

$$= \frac{-\log x}{1+x} + \log|x| - \log|1+x| + C$$

$$= \frac{-\log x}{1+x} + \log\left|\frac{x}{1+x}\right| + C$$

ଉଦାହରଣ 26.41

ମାନ ନିର୍ଣ୍ଣୟ କର : $\int x \cdot e^{2x} dx$

ସମାଧାନ :

$$\int x \cdot e^{2x} dx = x\frac{e^{2x}}{2} - \int 1 \cdot \frac{e^{2x}}{2} dx$$

$$= x \times \frac{e^{2x}}{2} - \frac{1}{2}\left(\frac{e^{2x}}{2}\right) + C$$

$$= \frac{x\ e^{2x}}{2} - \frac{1}{4}e^{2x} + C$$

ଉଦାହରଣ 26.42

ମାନ ନିର୍ଣ୍ଣୟ କର : $\int \sin^{-1} x\ dx$

ସମାଧାନ :

$$\int \sin^{-1} x\ dx = \int \sin^{-1} x \cdot 1 \cdot dx$$

$$= x\sin^{-1} x - \int \frac{x}{\sqrt{1-x^2}} dx$$

$1 - x^2 = t$ ନିଆଯାଉ । ତେବେ $-2x\ dx = dt$

$$\Rightarrow x\ dx = \frac{-1}{2} dt$$

$$\therefore \int \frac{x}{\sqrt{1-x^2}} dx = -\frac{1}{2}\int \frac{dt}{\sqrt{t}} = -\sqrt{t} + c = -\sqrt{1-x^2} + C$$

$$\therefore \int \sin^{-1} x\, dx = x\sin^{-1} x + \sqrt{1-x^2} + C$$

ଆସ ନିଜେ ନିଜକୁ ପରଖିବା 26.6

ମାନ ନିର୍ଣ୍ଣୟ କର :

1. (a) $\int x \sin x\, dx$ (b) $\int (1+x^2) \cos 2x\, dx$ (c) $\int x \sin 2x\, dx$

2. (a) $\int x \tan^2 x\, dx$ (b) $\int x^2 \sin^2 x\, dx$

3. (a) $\int x^3 \log 2x\, dx$ (b) $\int (1-x^2) \log x\, dx$ (c) $\int (\log x)^2\, dx$

4. (a) $\int \frac{\log x}{x^n}\, dx$ (b) $\int \frac{\log(\log x)}{x}\, dx$

5. (a) $\int x^2 e^{3x}\, dx$ (b) $\int x\, e^{3x}\, dx$

6. $\int x\, (\log x)^2\, dx$

7. (a) $\int \sec^{-1} x\, dx$ (b) $\int x \cot^{-1} x\, dx$

26.7 □ex [f(x) + f ′(x)] dx ରୂପର ସମାକଳନ

$$\int e^x [f(x) + f'(x)]\, dx$$

ଯେଉଁଠି f′(x) ହେଉଛି f(x)ର ଅବକଳନ । ଏପରି ସମାକଳନ କ୍ଷେତ୍ରରେ ଖଣ୍ଡଶଃ ପ୍ରଣାଳୀ ଦ୍ୱାରା ସମାକଳନ କଲାବେଳେ ସମାଧାନ ହେଉଛି $e^x(f(x)) + c$

ଉଦାହରଣସ୍ୱରୂପ, $\int e^x [\tan x + \log \sec x]\, dx$ ... (1) ସମ୍ବନ୍ଧରେ ବିଚାର କରାଯାଉ ।

ମନେକରାଯାଉ $f(x) = \log \sec x$

ତେବେ $f'(x) = \frac{\sec x \tan x}{\sec x} = \tan x$

ଏଣୁ (1)କୁ ଲେଖିବା

$$\int e^x [f'(x) + f(x)]\, dx = e^x (f(x)) + c = e^x \log \sec x + c$$

ବିକଳ୍ପ ଭାବରେ, ତୁମେ ଏହାର ମାନ ନିମ୍ନମତେ ନିର୍ଣ୍ଣୟ କରିପାର :

$$\int e^x [\tan x + \log \sec x]\, dx = \int \underset{\text{୧ମ}}{e^x} \underset{\text{୨ୟ}}{\tan x} + \int e^x \log \sec x\, dx$$

$$= e^x \log \sec x - \int e^x \log \sec dx + \int e^x \log \sec x\, dx$$

$$= e^x \log \sec x + C$$

ଉଦାହରଣ 26.43

ନିମ୍ନଲିଖିତ ପ୍ରତ୍ୟେକର ମାନ ନିର୍ଣ୍ଣୟ କର :

(a) $\int e^x \left(\frac{1}{x} - \frac{1}{x^2}\right) dx$ (b) $\int e^x \left(\frac{1 + x \log x}{x}\right) dx$

(c) $\int \frac{x e^x}{(x+1)^2}\, dx$ (d) $\int e^x \left[\frac{1+\sin x}{1+\cos x}\right] dx$

ସମାଧାନ : (a) $\int e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)dx = \int e^x\left[\frac{1}{x}+\frac{d}{dx}\left(\frac{1}{x}\right)\right]dx = e^x\left(\frac{1}{x}\right)$

(b) $$\int e^x\left(\frac{1+x\log x}{x}\right)dx = \int e^x\left(\frac{1}{x}+\log x\right)dx$$

$$= \int e^x\left(\log x+\frac{d}{dx}(\log x)\right)dx$$

$$= e^x \log x + C$$

(c) $$\int \frac{xe^x}{(x+1)^2}dx = \int \frac{x+1-1}{(x+1)^2}e^x\,dx$$

$$= \int e^x\left(\frac{1}{x+1}-\frac{1}{(x+1)^2}\right)dx$$

$$= \int e^x\left(\frac{1}{x+1}+\frac{d}{dx}\left(\frac{1}{(x+1)}\right)\right)dx$$

$$= e^x\left(\frac{1}{x+1}\right)+C$$

(d) $$\int e^x\left[\frac{1+\sin x}{1+\cos x}\right]dx = \int e^x\left[\frac{1+2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right]dx$$

$$= \int e^x\left[\frac{1}{2}\sec^2\frac{x}{2}+\tan\frac{x}{2}\right]dx$$

$$= \int e^x\left[\tan\frac{x}{2}+\frac{d}{dx}\left(\tan\frac{x}{2}\right)\right]dx$$

$$= e^x\tan\frac{x}{2}+C$$

ଉଦାହରଣ 26.44

ନିମ୍ନସ୍ଥ ପ୍ରତ୍ୟେକର ମାନ ନିର୍ଣ୍ଣୟ କର :

(a) $\int \sec^3 x\,dx$ (b) $\int e^x \sin x\,dx$

ସମାଧାନ : (a) $\int \sec^3 x\,dx$

ମନେକରାଯାଉ $I = \int \sec x\cdot\sec^2 x\,dx$

$$= \sec x\cdot\tan x - \int \sec x\ \tan x\cdot\tan x\,dx$$

$\therefore I = \sec x \tan x - \int (\sec^3 x - \sec x)\, dx \qquad [\because \tan^2 x = \sec^2 x - 1]$

ବା, $I = \sec x \tan x - \int \sec^3 x\, dx + \int \sec x\, dx$

ବା, $2I = \sec x \tan x + \int \sec x\, dx$

ବା, $2I = \sec x \tan x + \log|\sec x + \tan x| + C$

ବା, $I = \frac{1}{2}[\sec x \tan x + \log|\sec x] + \tan x| + C$

(b) $\int e^x \sin x\, dx$

ମନେକରାଯାଉ $I = \int e^x \sin x\, dx$

$= e^x(-\cos x) - \int e^x(-\cos x)\, dx$

$= -e^x \cos x + \int e^x \cos x\, dx$

$= -e^x \cos x + \left(e^x \sin x - \int e^x \sin x\, dx\right)$

$\therefore I = -e^x \cos x + e^x \sin x - I$

ବା, $2I = -e^x \cos x + e^x \sin x$

ବା, $I = \frac{e^x}{2}(\sin x - \cos x) + C$

ଉଦାହରଣ 26.45

ମାନ ନିର୍ଣ୍ଣୟ କର : $\int \sqrt{a^2 - x^2}\, dx$

ସମାଧାନ : ମନେକରାଯାଉ $I = \int \sqrt{a^2 - x^2}\, dx = \int \sqrt{a^2 - x^2} \cdot 1\, dx$

ଖଣ୍ଡଶଃ ସମାକଳନ କଲାବେଳେ 1କୁ ଦ୍ୱିତୀୟ ଫଳନ ନେଲେ, ପାଇବା

$$I = \left(\sqrt{a^2 - x^2}\right)x - \int \frac{1}{2\sqrt{a^2 - x^2}}(-2x) \cdot x\, dx$$

$$= x\sqrt{a^2 - x^2} + \int \frac{x^2}{\sqrt{a^2 - x^2}}\, dx$$

$$= x\sqrt{a^2 - x^2} + \int \frac{a^2 - (a^2 - x^2)}{\sqrt{a^2 - x^2}}\, dx$$

$$= x\sqrt{a^2 - x^2} + a^2 \int \frac{1}{\sqrt{a^2 - x^2}}\, dx - \int \sqrt{a^2 - x^2}\, dx$$

$$\therefore I = x\sqrt{a^2 - x^2} + a^2 \sin^{-1}\left(\frac{x}{a}\right) - I$$

ବା, $2I = x\sqrt{a^2 - x^2} + a^2 \sin^{-1}\left(\frac{x}{a}\right)$

ବା, $I = \frac{1}{2}\left[x\sqrt{a^2 - x^2} + a^2 \sin^{-1}\left(\frac{x}{a}\right)\right] + C$

ସେହିପରି, $\int \sqrt{x^2 - a^2}\, dx = \frac{x\sqrt{x^2 - a^2}}{2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2 - a^2}\right| + c$

$\therefore \int \sqrt{a^2 + x^2}\, dx = \frac{x\sqrt{a^2 + x^2}}{2} - \frac{a^2}{2}\log\left|x + \sqrt{a^2 + x^2}\right| + C$

ଉଦାହରଣ 26.46 ମାନ ନିର୍ଣ୍ଣୟ କର :

(a) $\int \sqrt{16x^2 + 25}\, dx$ (b) $\int \sqrt{16 - x^2}\, dx$ (c) $\int \sqrt{1 + x - 2x^2}\, dx$

ସମାଧାନ : (a) $\int \sqrt{16x^2 + 25}\, dx = 4\int \sqrt{x^2 + \frac{25}{16}}\, dx = 4\int \sqrt{x^2 + \left(\frac{5}{4}\right)^2}\, dx$

$\int \sqrt{(x^2 + a^2)}\, dx$ ର ସୂତ୍ର ପ୍ରୟୋଗ କରି ପାଇବା,

$$\int \sqrt{16x^2 + 25}\, dx = \left[\frac{x}{2}\sqrt{x^2 + \frac{25}{16}} + \frac{25}{32}\log\left|x + \sqrt{x^2 + \frac{25}{16}}\right|\right] + c$$

$$= \frac{x}{8}\sqrt{16x^2 + 25} + \frac{25}{8}\log\left|4x + \sqrt{16x^2 + 25}\right| + C$$

(b) $\int \sqrt{(a^2 - x^2)}\, dx$ ର ସୂତ୍ର ପ୍ରୟୋଗ କରି ପାଇବା,

$$\int \sqrt{16 - x^2}\, dx = \int \sqrt{(4)^2 - x^2}\, dx$$

$$= \frac{x}{2}\sqrt{16 - x^2} + \frac{16}{2}\sin^{-1}\frac{x}{4} + C$$

(c) $\int \sqrt{1 + x - 2x^2}\, dx = \sqrt{2}\int \sqrt{\frac{1}{2} + \frac{x}{2} - x^2}\, dx$

$$= \sqrt{2}\int \sqrt{\left[\frac{1}{2} - \left(x^2 - \frac{x}{2} + \frac{1}{16}\right) + \frac{1}{16}\right]}\, dx$$

$$= \sqrt{2}\int \sqrt{\left(\frac{3}{4}\right)^2 - \left(x - \frac{1}{4}\right)^2}\, dx$$

$$= \sqrt{2}\left[\frac{x - \frac{1}{4}}{2}\sqrt{\frac{9}{16} - \left(x - \frac{1}{4}\right)^2} + \frac{9}{16 \times 2}\sin^{-1}\frac{x - \frac{1}{4}}{\frac{3}{4}}\right] + c$$

$$= \sqrt{2}\left[\frac{4x-1}{8}\cdot\frac{1}{\sqrt{2}}\sqrt{1+x-2x^2}+\frac{9}{32}\sin^{-1}\frac{4x-1}{3}\right]+C$$

$$= \frac{4x-1}{8}\sqrt{1+x-2x^2}+\frac{9\sqrt{2}}{32}\sin^{-1}\frac{4x-1}{3}+C$$

ଆସ ନିଜେ ନିଜକୁ ପରଖିବା 26.7

ମାନ ନିର୍ଣ୍ଣୟ କର :

1. (a) $\int e^x \sec x[1+\tan x]dx$ (b) $\int e^x[\sec x+\log|\sec x+\tan x|]dx$

2. (a) $\int \frac{x-1}{x^2}e^x dx$ (b) $\int e^x\left(\sin^{-1}x-\frac{1}{\sqrt{1-x^2}}\right)dx$

3. $\int e^x \frac{x-1}{(x+1)^3}dx$ 4. $\int \frac{x e^x}{(x+1)^2}dx$

5. $\int \frac{x+\sin x}{1+\cos x}dx$ 6. $\int e^x \sin 2x\, dx$

26.8 ଆଂଶିକ ଭଗ୍ନାଂଶର ବ୍ୟବହାର ଦ୍ୱାରା ସମାକଳନ

ବର୍ତ୍ତମାନ ସୁଦ୍ଧା ସମାକଳନ ଲାଗି ତୁମେ ବିଭିନ୍ନ କୌଶଳ ଶିଖିଲଣି । ଆହୁରି ମଧ୍ୟ $\frac{4x+5}{x^2+x-6}$ ଭଳି କେତେକ କ୍ଷେତ୍ରରେ ପ୍ରତିସ୍ଥାପନ ବା ଭାଗ ଭାଗ ସମାକଳନ ପଦ୍ଧତି ଆମକୁ ସାହାଯ୍ୟ କରିବ ନାହିଁ । ଏପରି କ୍ଷେତ୍ରରେ ଆମେ ଅନ୍ୟ ଏକ କୌଶଳର ଉପଯୋଗ କରିବା, ତାହା ହେଉଛି **ଆଂଶିକ ଭଗ୍ନାଂଶ ପ୍ରକ୍ରିୟା ବ୍ୟବହାର କରି ସମାକଳନ କରିବା ।**

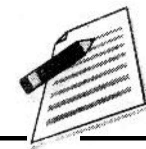
ଯେକୌଣସି ପ୍ରକୃତ ପରିମେୟ ଭଗ୍ନାଂଶ $\frac{p(x)}{q(x)}$ କୁ ପରିମେୟ ଭଗ୍ନାଂଶମାନଙ୍କର ସମଷ୍ଟି ରୂପେ ପ୍ରକାଶ କରିହେବ ଯେପରିକି ପ୍ରତ୍ୟେକ ପରିମେୟ ଭଗ୍ନାଂଶରେ କେବଳ ଗୋଟିଏ ଉତ୍ପାଦକ $q(x)$ ଥିବ । ଏଭଳି ପ୍ରତ୍ୟେକ ଭଗ୍ନାଂଶକୁ **ଆଂଶିକ ଭଗ୍ନାଂଶ** କୁହାଯାଏ ଏବଂ ଏହି ପ୍ରକ୍ରିୟାକୁ ବିଶ୍ଳେଷଣ ବା ପ୍ରକ୍ରିୟା ଦ୍ୱାରା ଦତ୍ତ ଭଗ୍ନାଶଂକୁ ଆଂଶିକ ଭଗ୍ନାଂଶରେ ପରିଣତ କରିବା ପ୍ରକ୍ରିୟା ବୋଲି କୁହାଯାଏ ।

ଉଦାହରଣସ୍ୱରୂପ, $\frac{3}{x+2}+\frac{5}{x-1}=\frac{8x+7}{(x+2)(x-1)}=\frac{8x+7}{x^2+x-2}$

ଏଠାରେ $\frac{3}{x+2}, \frac{5}{x-1}$ କୁ $\frac{8x+7}{x^2+x-2}$ ର ଆଂଶିକ ଭଗ୍ନାଂଶ କୁହାଯାଏ ।

ଯଦି $\frac{f(x)}{g(x)}$ ଏକ ପ୍ରକୃତ ଭଗ୍ନାଂଶ ହୋଇଥାଏ ଏବଂ $g(x)$ କୁ ବାସ୍ତବ ଉତ୍ପାଦକମାନଙ୍କର ଗୁଣଫଳ ରୂପେ ପ୍ରକାଶ କରିବା ସମ୍ଭବ ହୋଇଥାଏ, ତେବେ

(a) ପ୍ରତ୍ୟେକ ଅଣପୌନଃପୁନିକ (ଏକାଧିକ ବାର ନ ଥିବା) ଏକଘାତୀ ଉତ୍ପାଦକ $ax + b$ ଲାଗି ଗୋଟିଏ ଆଂଶିକ ଭଗ୍ନାଂଶ ରହିବ ଯାହାର ରୂପ ହେବ



$$\frac{A}{ax+b}$$

(b) $(ax+b)^2$ ଲାଗି ଆମେ ଦୁଇଟି ଆଂଶିକ ଭଗ୍ନାଂଶର ସମଷ୍ଟି ନେଇଥାଉ, ଯାହାର ରୂପ ହୁଏ

$$\frac{A}{ax+b}+\frac{B}{(ax+b)^2}$$

$(ax+b)^3$ ଲାଗି ଆମେ ତିନୋଟି ଆଂଶିକ ଭଗ୍ନାଂଶର ସମଷ୍ଟି ନେଇଥାଉ, ଯାହାର ରୂପ ହୁଏ

$$\frac{A}{ax+b}+\frac{B}{(ax+b)^2}+\frac{C}{(ax+b)^3}$$ ଇତ୍ୟାଦି ।

(c) ଉତ୍ପାଦକରେ ବିଶ୍ଳେଷଣ ହୋଇପାରୁନଥିବା ଦ୍ୱିଘାତୀ ପଲିନୋମିଆଲ୍ ax^2+bx+c କ୍ଷେତ୍ରରେ ଆମେ ଯେଉଁ ଆଂଶିକ ଭଗ୍ନାଂଶ ପାଇଁ ତାହା ହେଉଛି

$$\frac{Ax+B}{ax^2+bx+c}$$

ଏଣୁ କୌଣସି ପ୍ରକୃତ ଭଗ୍ନାଂଶ $\frac{f(x)}{g(x)}$ ରେ $g(x)$କୁ ଯଦି ଏକାଧିକ ଉତ୍ପାଦକରେ ପରିଣତ କରାଯାଇ ପାରୁଥାଏ, ତେବେ $\frac{f(x)}{g(x)}$ କୁ ନିମ୍ନ ରୂପେ ଲେଖାଯାଇପାରେ :

ହରରେ ଥିବା ଉତ୍ପାଦକ	ସମ୍ପୃକ୍ତ ଆଂଶିକ ଭଗ୍ନାଂଶ
$ax+b$	$\frac{A}{ax+b}$
$(ax+b)^2$	$\frac{A}{ax+b}+\frac{B}{(ax+b)^2}$
$(ax+b)^3$	$\frac{A}{ax+b}+\frac{B}{(ax+b)^2}+\frac{C}{(ax+b)^3}$
ax^2+bx+c	$\frac{Ax+B}{ax^2+bx+c}$
$(ax^2+bx+c)^2$	$\frac{Ax+B}{ax^2+bx+c}+\frac{Cx+D}{(ax^2+bx+c)^2}$

ଯେଉଁଠି A, B, C ଓ D ହେଉଛନ୍ତି ଯାଦୃଚ୍ଛିକ ଧ୍ରୁବକ ।

ଯେଉଁ ପରିମେୟ ଭଗ୍ନାଂଶମାନଙ୍କର ହରକୁ ଏକଘାତୀ ବା ଦ୍ୱିଘାତୀ ଉତ୍ପାଦକମାନଙ୍କର ଗୁଣଫଳ ରୂପେ ପ୍ରକାଶ କରିହେବ, ସେହିମାନଙ୍କର ସମାକଳନ କଥା ଆମେ ବିଚାର କରିବା ।

ଉଦାହରଣ 26.47 ମାନ ନିର୍ଣ୍ଣୟ କର : $\int \frac{2x+5}{x^2-x-2}\,dx$

ସମାଧାନ : $\frac{2x+5}{x^2-x-2}=\frac{2x+5}{(x-2)(x+1)}$

ମନେକରାଯାଉ $\frac{2x+5}{(x-2)(x+1)}=\frac{A}{x-2}+\frac{B}{x+1}$ ... (1)

ଉଭୟପାର୍ଶ୍ୱକୁ (x–2) (x+1) ଦ୍ୱାରା ଗୁଣନ କଲେ, ପାଇବା

$2x + 5 = A(x+1) + B(x-2)$

$x = 2$ ନେଲେ, ପାଇବା $9 = 3A$ ବା $A = 3$

$x = -1$ ନେଲେ, ପାଇବା $3 = -3B$ ବା $B = -1$

ଏହି ମାନଗୁଡ଼ିକୁ (1)ରେ ବସାଇଲେ, ପାଇବା

$$\frac{2x+5}{(x-2)(x+1)} = \frac{3}{x-2} - \frac{1}{x+1}$$

$$\Rightarrow \int \frac{2x+5}{x^2-x-2}dx = \int \frac{3}{x-2}\,dx - \int \frac{1}{x+1}dx$$

$$= 3\log|x-2| - \log|x+1| + C$$

ଉଦାହରଣ 26.48 ମାନ ନିର୍ଣ୍ଣୟ କର :

(a) $\int \frac{x^2-x-1}{x^3-x^2-6x}\,dx$ (b) $\int \frac{1}{(x^2-1)(x+1)}dx$

ସମାଧାନ : (a) $\frac{x^2-x-1}{x^3-x^2-6x} = \frac{x^2-x-1}{x(x-3)(x+2)}$

ମନେକରାଯାଉ $\frac{x^2-x-1}{x(x-3)(x+2)} = \frac{A}{x} + \frac{B}{x-3} + \frac{C}{x+2}$ ... (1)

ଉଭୟପାର୍ଶ୍ୱକୁ x(x–3) (x+2) ଦ୍ୱାରା ଗୁଣନ କଲେ, ପାଇବା

$x^2 - x - 1 = A(x-3)(x+2) + Bx(x+2) + Cx(x-3)$

$x = 3$ ନେଲେ, ପାଇବା $15B = 5$ ବା $B = \frac{1}{3}$

$x = 0$ ନେଲେ, ପାଇବା $-6A = -1$ ବା $A = \frac{1}{6}$

$x = -2$ ନେଲେ, ପାଇବା $10C = 5$ ବା $C = \frac{1}{2}$

ଏହି ମାନଗୁଡ଼ିକୁ (1)ରେ ବସାଇଲେ, ପାଇବା

$$\frac{x^2-x-1}{x(x-3)(x+2)} = \frac{1}{6x} + \frac{1}{3(x+3)} + \frac{1}{2(x+2)}$$

$$\Rightarrow \int \frac{x^2-x-1}{x^3-x^2-6x}dx = \int \frac{1}{6x}dx + \int \frac{1}{3(x-3)}dx + \int \frac{1}{2(x+2)}dx$$

$$= \frac{1}{6}\log|x| + \frac{1}{3}\log|x-3| + \frac{1}{2}\log|x+2| + C$$

(b) $\frac{1}{(x^2-1)(x+1)} = \frac{1}{(x+1)(x-1)(x+1)} = \frac{1}{(x-1)(x+1)^2}$

ମନେକରାଯାଉ $\dfrac{1}{(x-1)(x+1)^2}=\dfrac{A}{x-1}+\dfrac{B}{x+1}+\dfrac{C}{(x+1)^2}$

ଉଭୟ ପାର୍ଶ୍ୱକୁ $(x^2-1)(x+1)$ ଦ୍ୱାରା ଗୁଣନ କଲେ, ଆମେ ପାଇବା

$1 = A(x+1)^2 + B(x-1)(x+1) + C(x-1)$

$x = 1$ ନେଲେ, ଆମେ ପାଇବା $A=\dfrac{1}{4}$

$x = -1$ ନେଲେ, ଆମେ ପାଇବା $C=\dfrac{-1}{2}$

$0 = A + B \Rightarrow B=-\dfrac{1}{4}$

$$\therefore \int \frac{1}{(x^2-1)(x+1)}\,dx=\int\frac{1}{4(x-1)}dx-\frac{1}{4}\int\frac{1}{x+1}dx-\frac{1}{2}\int\frac{1}{(x+1)^2}dx$$

$$=\frac{1}{4}\log|x-1|-\frac{1}{4}\log|x+1|-\frac{1}{2}\left[-\frac{1}{x+1}\right]+c$$

$$=\frac{1}{4}\log|x-1|-\frac{1}{4}\log|x+1|+\frac{1}{2(x+1)}+c$$

ଉଦାହରଣ 26.49

ମାନ ନିର୍ଣ୍ଣୟ କର :

$$\int\frac{x^3+x+1}{x^2-1}dx$$

ସମାଧାନ : ମନେକରାଯାଉ $I=\displaystyle\int\frac{x^3+x+1}{x^2-1}dx$

ବର୍ତ୍ତମାନ $\dfrac{x^3+x+1}{x^2-1}=x+\dfrac{2x+1}{x^2-1}=x+\dfrac{2x+1}{(x+1)(x-1)}$

$$\therefore I=\int\left(x+\frac{2x+1}{(x+1)(x-1)}\right)dx \qquad \ldots (1)$$

ମନେକରାଯାଉ $\dfrac{2x+1}{(x+1)(x-1)}=\dfrac{A}{x+1}+\dfrac{B}{x-1}$ ... (2)

$\Rightarrow 2x + 1 = A(x-1) + B(x+1)$

$x = 1$ ନେଲେ, ଆମେ ପାଇବା $B=\dfrac{3}{2}$

$x = -1$ ନେଲେ, ଆମେ ପାଇବା $A=\dfrac{1}{2}$

A ଓ Bର ମାନକୁ (2) ବସାଇ ଏବଂ ସମାକଳନ କରି ଆମେ ପାଇବା

$$\int \frac{2x+1}{x^2-1}\,dx = \frac{1}{2}\int \frac{1}{x+1}\,dx + \frac{3}{2}\int \frac{1}{x-1}\,dx$$

$$= \frac{1}{2}\log|x+1| + \frac{3}{2}\log|x-1| \qquad \ldots (3)$$

$\therefore$ (1) ଓ (3)ରୁ ଆମେ ପାଇବା

$$I = \frac{x^2}{2} + \frac{1}{2}\log|x+1| + \frac{3}{2}\log|x-1| + C$$

ଉଦାହରଣ 26.50

ମାନ ନିର୍ଣ୍ଣୟ କର :

(a) $\int \frac{8}{(x+2)(x^2+4)}\,dx$ (b) $\int \frac{1}{x(x^3+1)}\,dx$

ସମାଧାନ : (a) $\frac{8}{(x+2)(x^2+4)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+4}$

(x^2+4ର ଉତ୍ପାଦକୀକରଣ ସମ୍ଭବ ହୋଇନଥିବାରୁ)

ଉଭୟ ପାର୍ଶ୍ୱରେ $(x+2)(x^2+4)$ ଗୁଣନ କଲେ, ପାଇବା

$8 = A(x^2+4) + (Bx+C)(x+2)$

ଉଭୟ ପାର୍ଶ୍ୱରେ ଥିବା xର ଅନୁରୂପ ଘାତାଙ୍କ ବିଶିଷ୍ଟ ପଦଗୁଡ଼ିକର ସହଗମାନଙ୍କୁ ତୁଳନା କଲେ, ପାଇବା

$$\left.\begin{aligned} 0 &= A+B \\ 0 &= 2B+C \\ 8 &= 4A+2C \end{aligned}\right\} \Rightarrow A = 1,\ B = -1, C = 2$$

$$\therefore \int \frac{8}{(x+2)(x^2+4)}\,dx = \int \frac{1}{x+2}\,dx - \int \frac{x-2}{x^2-4}\,dx$$

$$= \int \frac{1}{x+2}\,dx - \frac{1}{2}\int \frac{2x}{x^2+4}\,dx + 2\int \frac{dx}{x^4+4}$$

$$= \log|x+2| - \frac{1}{2}\log|x^2+4| + 2\cdot\frac{1}{2}\tan^{-1}\frac{x}{2} + c$$

$$= \log|x+2| - \frac{1}{2}\log|x^2+4| + \tan^{-1}\frac{x}{2} + c$$

(b) $\frac{1}{x(x^3+1)} = \frac{1}{x(x+1)(x^2-x+1)} = \frac{A}{x} + \frac{B}{x+1} + \frac{Cx+D}{x^2-x+1}$

ଉଭୟ ପାର୍ଶ୍ୱକୁ $x(x^3+1)$ ବା $x(x+1)(x^2-x+1)$ ଦ୍ୱାରା ଗୁଣନ କଲେ, ପାଇବା

$1 = A(x+1)(x^2-x+1) + Bx(x^2-x+1) + (Cx+D)(x)(x+1)$

ଟିପ୍ପଣୀ

ଉଭୟ ପାର୍ଶ୍ୱରେ ଥିବା xର ଅନୁରୂପ ଘାତାଙ୍କ ବିଶିଷ୍ଟ ପଦଗୁଡ଼ିକର ସହଗମାନଙ୍କୁ ତୁଳନା କଲେ, ପାଇବା

$$\left.\begin{aligned} 0 &= A+B+C \\ 0 &= -B+C+D \\ 0 &= B+D \end{aligned}\right\} \Rightarrow A=1,\ B=-\frac{1}{3},\ C=-\frac{2}{3},\ D=\frac{1}{3}$$

$$1 = A$$

$$\therefore \frac{1}{x(x^3+1)} = \frac{1}{x} - \frac{1}{3(x+1)} - \frac{2x-1}{3(x^2-x+1)}$$

$$\therefore \int \frac{1}{x(x^3+1)}\,dx = \int \frac{1}{x}dx - \frac{1}{3}\int \frac{1}{x+1}dx - \frac{1}{3}\int \frac{2x-1}{x^2-x+1}dx$$

$$= \log|x| - \frac{1}{3}\log|x+1| - \frac{1}{3}\log|x^2-x+1| + C$$

ଉଦାହରଣ 26.51 ମାନ ନିର୍ଣ୍ଣୟ କର : $\int \frac{dx}{\sin x - \sin 2x}$

ସମାଧାନ : ମନେକରାଯାଉ $I = \int \frac{dx}{\sin x - \sin 2x} = \int \frac{dx}{\sin x - 2\sin x \cos x}$

$$= \int \frac{dx}{\sin x(1-2\cos x)}$$

ଲବ ଓ ହର ଉଭୟକୁ $\sin x$ ଦ୍ୱାରା ଗୁଣନ କଲେ, ପାଇବା

$$I = \int \frac{\sin x\,dx}{\sin^2 x(1-2\cos x)} = \int \frac{\sin x\,dx}{(1-\cos^2 x)(1-2\cos x)}$$

$$= \int \frac{\sin x\,dx}{(1+\cos x)(1-\cos x)(1-2\cos x)}$$

$\cos x = t$ ନିଅ, ତେବେ $-\sin x\,dx = dt$

$$\therefore I = \int \frac{-dt}{(1+t)(1-t)(1-2t)}$$

ମନେକରାଯାଉ, $\frac{-1}{(1+t)(1-t)(1-2t)} = \frac{A}{1-t} + \frac{B}{1+t} + \frac{C}{1-2t}$

ତେବେ, $-1 = A(1+t)(1-2t) + B(1-t)(1-2t) + C(1-t)(1+t)$

$t = -1$ ନେଲେ, ପାଇବା $B = -\frac{1}{6}$

$t = 1$ ନେଲେ, ପାଇବା $A = \frac{1}{2}$

$t = \frac{1}{2}$ ନେଲେ, ପାଇବା $C = -\frac{4}{3}$

$$\therefore I = \frac{1}{2}\int\frac{1}{1-t}dt - \frac{1}{6}\int\frac{1}{1+t}dt - \frac{4}{3}\int\frac{dt}{1-2t}$$

$$= -\frac{1}{2}\log|1-t| - \frac{1}{6}\log|1+t| + \frac{4}{6}\log|1-2t| + C$$

$$= -\frac{1}{2}\log|1-\cos x| - \frac{1}{6}\log|1+\cos x| + \frac{2}{3}\log|1-2\cos x| + C$$

ଉଦାହରଣ 26.52

ମାନ ନିର୍ଣ୍ଣୟ କର : $\int\frac{2\sin 2\theta - \cos\theta}{4-\cos^2\theta - 4\sin\theta}d\theta$

ସମାଧାନ : ମନେକରାଯାଉ $I = \int\frac{2\sin 2\theta - \cos\theta}{4-\cos^2\theta - 4\sin\theta}d\theta$

$$= \int\frac{(4\sin\theta - 1)\cos\theta\, d\theta}{3+\sin^2\theta - 4\sin\theta}$$

ମନେକରାଯାଉ, $\sin\theta = t$, ତେବେ $\cos\theta\, d\theta = dt$

$$\therefore I = \int\frac{4t-1}{3+t^2-4t}dt$$

ମନେକରାଯାଉ $\frac{4t-1}{3+t^2-4t} = \frac{A}{t-3} + \frac{B}{t-1}$

ଫଳରେ, $4t - 1 = A(t-1) + B(t-3)$

$t = 1$ ନିଅ, ତେବେ $B = -\frac{3}{2}$

$t = 3$ ନିଅ, ତେବେ $A = \frac{11}{2}$

$$\therefore I = \frac{11}{2}\int\left(\frac{1}{(t-3)}\right)dt - \frac{3}{2}\int\frac{dt}{t-1}$$

$$= \frac{11}{2}\log|t-3| - \frac{3}{2}\log|t-1| + C$$

$$= \frac{11}{2}\log|\sin\theta - 3| - \frac{3}{2}\log|\sin\theta - 1| + C$$

ଉଦାହରଣ 26.53

ମାନ ନିର୍ଣ୍ଣୟ କର : $\int\frac{\tan\theta + \tan^3\theta}{1+\tan^3\theta}d\theta$

ସମାଧାନ : ମନେକରାଯାଉ $I = \int\frac{\tan\theta + \tan^3\theta}{1+\tan^3\theta}d\theta = \int\frac{\tan\theta(1+\tan^2\theta)}{1+\tan^3\theta}d\theta = \int\frac{\tan\theta\cdot\sec^2\theta}{1+\tan^3\theta}d\theta$

ମନେକରାଯାଉ $\tan\theta = t$, ତେବେ $\sec^2\theta\, d\theta = dt$

$$\therefore I = \int \frac{t\,dt}{1+t^3} = \int \frac{t\,dt}{(1+t)(1-t+t^2)}$$

ମନେକରାଯାଉ $\dfrac{t}{(1+t)(1-t+t^2)} = \dfrac{A}{1+t} + \dfrac{Bt+C}{1-t+t^2}$

ଫଳରେ, $t = A(1-t+t^2) + (Bt+C)(1+t)$

ଉଭୟ ପାର୍ଶ୍ୱରେ ଥିବା tର ସମାନ ଘାତ ବିଶିଷ୍ଟ ପଦଗୁଡ଼ିକୁ ତୁଳନା କଲେ, ପାଇବା

$A + B = 0, -A + B + C = 1, A + C = 0$

$$\Rightarrow A = -\frac{1}{3}, B = \frac{1}{3}, C = \frac{1}{3}$$

$$\therefore I = -\frac{1}{3}\int \frac{1}{1+t}dt + \frac{1}{3}\int \frac{t+1}{1-t+t^2}dt$$

$$= -\frac{1}{3}\int \frac{dt}{1+t} + \frac{1}{6}\int \frac{2t+2}{t^2-t+1}dt$$

$$= -\frac{1}{3}\int \frac{dt}{1+t} + \frac{1}{6}\int \frac{(2t-1)+3}{t^2-t+1}dt$$

$$= -\frac{1}{3}\int \frac{dt}{1+t} + \frac{1}{6}\int \frac{(2t-1)\,dt}{t^2-t+1} + \frac{1}{2}\int \frac{1\,dt}{\left(t-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= -\frac{1}{3}\log|1+t| + \frac{1}{6}\log|t^2-t+1| + \frac{1}{2}\cdot\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{t-\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)$$

$$= -\frac{1}{3}\log|1+t| + \frac{1}{6}\log|t^2-t+1| + \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2t-1}{\sqrt{3}}\right) + c$$

$$= -\frac{1}{3}\log|1+\tan\theta| + \frac{1}{6}\log|\tan^2\theta - \tan\theta$$

$$+1\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2\tan\theta-1}{\sqrt{3}}\right) + c$$

ଆସ ନିଜେ ନିଜକୁ ପରଖିବା 26.8

ମାନ ନିର୍ଣ୍ଣୟ କର :

1. (a) $\int \sqrt{4x^2-5}\,dx$ (b) $\int \sqrt{x^2+3x}\,dx$ (c) $\int \sqrt{3-2x-2x^2}\,dx$

2. (a) $\int \frac{x+1}{(x-2)(x-3)}dx$ (b) $\int \frac{x}{x^2-16}dx$

3. (a) $\int \frac{x^3}{x^2-4}dx$ (b) $\int \frac{2x^2+x+1}{(x-1)^2(x+2)}dx$

4. $\int \frac{x^2+x+1}{(x-1)^3}dx$

5. (a) $\int \frac{\sin x}{\sin 4x}dx$ (b) $\int \frac{1-\cos x}{\cos x(1+\cos x)}dx$

ଆମେ ଯାହା ଶିଖିଲେ :

- ସମାକଳନ ହେଉଛି ଅବକଳନର ପ୍ରତିଲୋମ ପ୍ରକ୍ରିୟା
- କେତେକ ଅନିର୍ଦ୍ଦିଷ୍ଟ ସମାକଳନର ମାନକ ରୂପ

(a) $\int x^n\,dx = \frac{x^{n+1}}{n+1}+C \qquad (n \neq -1)$

(b) $\int \frac{1}{x}\,dx = \log|x|+C$

(c) $\int \sin x\,dx = -\cos x + C$

(d) $\int \cos x\,dx = \sin x + C$

(e) $\int \sec^2 x\,dx = \tan x + C$

(f) $\int \operatorname{cosec}^2 x\,dx = -\cot x + C$

(g) $\int \sec x \tan x\,dx = \sec x + C$

(h) $\int \operatorname{cosec} x \cot x\,dx = -\operatorname{cosec} x + C$

(i) $\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C$

(j) $\int \frac{1}{1+x^2}\,dx = \tan^{-1} x + C$

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(k) $\int \frac{1}{x\sqrt{x^2-1}}\,dx = \sec^{-1}x + C$

(l) $\int e^x\,dx = e^x + C$

(m) $\int a^x\,dx = \frac{a^x}{\log a} + C$ $(a > 0$ ଏବଂ $a \neq 1)$

- ଅନିର୍ଦ୍ଦିଷ୍ଟ ସମାକଳମାନଙ୍କର ଧର୍ମ

(a) $\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$

(b) $\int kf(x)\,dx = k\int f(x)\,dx$

- (i) $\int (ax+b)^n\,dx = \frac{1}{a}\frac{(ax+b)^{n+1}}{n+1} + C$ $\qquad (n \neq -1)$

(ii) $\int \frac{1}{ax+b}\,dx = \frac{1}{a}\log|ax+b| + C$

(iii) $\int \sin(ax+b)\,dx = \frac{-1}{a}\cos(ax+b) + C$

(iv) $\int \cos(ax+b)\,dx = \frac{1}{a}\sin(ax+b) + C$

(v) $\int \sec^2(ax+b)\,dx = \frac{1}{a}\tan(ax+b) + C$

(vi) $\int \operatorname{cosec}^2(ax+b)\,dx = -\frac{1}{a}\cot(ax+b) + C$

(vii) $\int \sec(ax+b)\tan(ax+b)\,dx = \frac{1}{a}\sec(ax+b) + C$

(viii) $\int \operatorname{cosec}(ax+b)\cot(ax+b)\,dx = -\frac{1}{a}\operatorname{cosec}(ax+b) + C$

(ix) $\int e^{ax+b}\,dx = \frac{1}{a}e^{ax+b} + C$

(x) $\int \frac{f'(x)}{f(x)}\,dx = \log|f(x)| + C$

- (i) $\int \tan x\,dx = -\log|\cos x| + C = \log|\sec x| + C$

(ii) $\int \cot x\,dx = \log|\sin x| + C$

(iii) $\int \sec x\,dx = \log|\sec x + \tan x| + C$

(iv) $\int \operatorname{cosec} x\,dx = \log|\operatorname{cosec} x - \cot x| + C$

- (i) $\int \frac{1}{a^2 - x^2}\, dx = \frac{1}{2a} \log\left|\frac{a+x}{a-x}\right| + c$

  (ii) $\int \frac{1}{a^2 - x^2}\, dx = \frac{1}{2a} \log\left|\frac{x-a}{x+a}\right| + c$

  (iii) $\int \frac{1}{x^2 + a^2}\, dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$

  (iv) $\int \frac{1}{\sqrt{a^2 - x^2}}\, dx = \sin^{-1} \frac{x}{a} + c$

  (v) $\int \frac{1}{\sqrt{x^2 - a^2}}\, dx = \log\left|x + \sqrt{x^2 - a^2}\right| + c$

  (vi) $\int \frac{dx}{\sqrt{x^2 + a^2}} = \log\left|x + \sqrt{x^2 + a^2}\right| + c$

- (i) $\int \frac{x^2 + 1}{x^4 + 1} dx = \frac{1}{2\sqrt{2}} \tan^{-1}\left(\frac{x - \frac{1}{x}}{\sqrt{2}}\right) + c$

  (ii) $\int \frac{x^2 - 1}{x^4 + 1} dx = \frac{1}{2\sqrt{2}} \log\left|\frac{x + \frac{1}{x} - \sqrt{2}}{x + \frac{1}{x} + \sqrt{2}}\right| + c$

  (iii) $\int \frac{x^2}{x^4 + 1} dx = \frac{1}{2\sqrt{2}}\left[\tan^{-1}\left(\frac{x - \frac{1}{2}}{\sqrt{2}}\right) + \log\left|\frac{x + \frac{1}{x} - \sqrt{2}}{x + \frac{1}{x} + \sqrt{2}}\right|\right] + c$

  (iv) $\int \frac{1}{x^4 + 1} dx = \frac{1}{2\sqrt{2}} \tan^{-1}\left(\frac{x - \frac{1}{2}}{\sqrt{2}}\right) - \frac{1}{2\sqrt{2}} \log\left|\frac{x + \frac{1}{x} - \sqrt{2}}{x + \frac{1}{x} + \sqrt{2}}\right| + c$

- (i) $\int \frac{x^2 + 1}{x^4 + x^2 + 1} dx = \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{x - \frac{1}{x}}{\sqrt{3}}\right) + c$

  (ii) $\int \frac{x^2 - 1}{x^4 + x^2 + 1} dx = \frac{1}{2} \log\left|\frac{x + \frac{1}{x} - 1}{x + \frac{1}{x} + 1}\right| + c$

  (iii) $\int \frac{1}{x^4 + x^2 + 1}\, dx = \frac{1}{2}\left[\frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{\tan x - 1}{\sqrt{2}}\right) + \frac{1}{\sqrt{2}} \tan^{-1} + \left(\frac{\cot x - 1}{\sqrt{2}}\right) + c\right.$

  (iv) $\int \left(\sqrt{\tan x} + \sqrt{\cot x}\right) dx = \sqrt{2} \sin^{-1} (\sin x - \cos x) + c$

- ଦୁଇଟି ଫଳନର ଗୁଣଫଳର ସମାକଳ

  ପ୍ରଥମ ଫଳନ × ଦ୍ୱିତୀୟ ଫଳନର ସମାକଳ − ∫ [ପ୍ରଥମ ଫଳନର ଅବକଳଜ × ଦ୍ୱିତୀୟ ଫଳନର ସମାକଳ]dx

- $\int e^x [f(x)+f'(x)]dx = e^x f(x)+c$

  $$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2}+a^2 \sin^{-1}\left(\frac{x}{a}\right)\right]+c$$

  $$\int \sqrt{x^2-a^2}\, dx = \frac{x\sqrt{x^2-a^2}}{2}-\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+c$$

  $$\int \sqrt{a^2+x^2}\, dx = \frac{x\sqrt{a^2+x^2}}{2}+\frac{a^2}{2}\log\left|x+\sqrt{a^2+x^2}\right|+c$$

- ପରିମେୟ ଫଳନ ନିମ୍ନସ୍ଥ ଦୁଇପ୍ରକାର ହୋଇଥାଏ ।

  (i) ପ୍ରକୃତ, ଯେଉଁଠି ଲବରେ ଥିବା ଚଳର ଘାତାଙ୍କ < ହରରେ ଥିବା ଚଳର ଘାତାଙ୍କ ।

  (ii) ଅପ୍ରକୃତ, ଯେଉଁଠି ଲବରେ ଥିବା ଚଳର ଘାତାଙ୍କ ≥ ହରରେ ଥିବା ଚଳର ଘାତାଙ୍କ ।

- ଯଦି ପ୍ରକୃତ ପରିମେୟ ଫଳନ $\frac{f(x)}{g(x)}$ ର $g(x)$କୁ ବାସ୍ତବ ଉତ୍ପାଦକ ମାନଙ୍କର ଗୁଣଫଳ ରୂପେ ପ୍ରକାଶ କରାଯାଇପାରେ, ତେବେ $\frac{f(x)}{g(x)}$ କୁ ନିମ୍ନରୂପରେ ଲେଖିବା ସମ୍ଭବ ହୁଏ :

ହରରେ ଥିବା ଉତ୍ପାଦକ	**ସମ୍ପୃକ୍ତ ଆଂଶିକ ଭଗ୍ନାଂଶ**
$ax + b$	$\frac{A}{ax+b}$
$(ax+b)^2$	$\frac{A}{ax+b}+\frac{B}{(ax+b)^2}$
$(ax+b)^3$	$\frac{A}{ax+b}+\frac{B}{(ax+b)^2}+\frac{C}{(ax+b)^3}$
$ax^2 + bx + c$	$\frac{Ax+B}{ax^2+bx+c}$
$(ax^2+bx+c)^2$	$\frac{Ax+B}{ax^2+bx+c}+\frac{Cx+D}{(ax^2+bx+c)^2}$

ଯେଉଁଠି A, B, C, D ହେଉଛନ୍ତି ଯାଦୃଚ୍ଛିକ ଧ୍ରୁବକ ।

ସହାୟକ ୱେବ୍‌ସାଇଟ୍

- *http://www.wikipedia.org*
- *http://mathworld.wolfram.com*

ମଡ୍ୟୁଲ-V
ଫଳନ

ଟିପ୍ପଣୀ

ପାଠ ଶେଷ ଅଭ୍ୟାସ କାର୍ଯ୍ୟ

ନିମ୍ନ ଫଳନଗୁଡ଼ିକର x ସାପେକ୍ଷ ସମାକଳନ କର :

1. $\dfrac{\sin^3 x+\cos^3 x}{\sin^2 x\cos^2 x}$

2. $\sqrt{1+\sin 2x}$

3. $\dfrac{\cos 2x}{\cos^2 x\sin^2 x}$

4. $(\tan x-\cot x)^2$

5. $\dfrac{4}{1+x^2}-\dfrac{1}{\sqrt{1-x^2}}$

6. $\dfrac{2\sin^2 x}{1+\cos 2x}$

7. $\dfrac{2\cos^2 x}{1-\cos 2x}$

8. $\left(\sin\dfrac{x}{2}+\cos\dfrac{x}{2}\right)^2$

9. $\left(\cos\dfrac{x}{2}-\sin\dfrac{x}{2}\right)^2$

10. $\cos(7x-\pi)$

11. $\sin(3x+4)$

12. $\sec^2(2x+b)$

13. $\displaystyle\int\frac{dx}{\sin x-\cos x}$

14. $\displaystyle\int\frac{1}{(1+x^2)\tan^{-1}x}dx$,

15. $\displaystyle\int\frac{\operatorname{cosec} x}{\log\left(\tan\frac{x}{2}\right)}dx$

16. $\displaystyle\int\frac{\cot x}{3+4\log\sin x}dx$

17. $\displaystyle\int\frac{dx}{\sin 2x\log\tan x}$

18. $\displaystyle\int\frac{e^x+1}{e^x-1}dx$

19. $\displaystyle\int\sec^4 x\tan x\,dx$

20. $\displaystyle\int e^x\sin e^x\,dx$

21. $\displaystyle\int\frac{x\,dx}{\sqrt{2x^2+3}}$

22. $\displaystyle\int\frac{\sec^2 x}{\sqrt{\tan x}}dx$

23. $\displaystyle\int\sqrt{25-9x^2}\,dx$

24. $\displaystyle\int\sqrt{2ax-x^2}\,dx$

25. $\displaystyle\int\sqrt{3x^2+4}\,dx$

26. $\displaystyle\int\sqrt{1+9x^2}\,dx$

27. $\displaystyle\int\frac{x^2\,dx}{\sqrt{x^2-a^2}}$

28. $\displaystyle\int\frac{dx}{\sin^2 x+4\cos^2 x}$

29. $\displaystyle\int\frac{dx}{2+\cos x}$

30. $\displaystyle\int\frac{dx}{x^2-6x+13}$

31. $\displaystyle\int\frac{dx}{1+3\sin^2 x}$

32. $\displaystyle\int\frac{x^2}{x^2-a^2}dx$

33. $\int \frac{dx}{x\sqrt{9+x^4}}$

34. $\int \frac{\sin x}{\sin 3x} dx$

35. $\int \frac{dx}{1-4\cos^2 x}$

36. $\int \sec^2(ax+b)\, dx$

37. $\frac{dx}{x(2+\log x)}$

38. $\int \frac{x^5}{1+x^6} dx$

39. $\int \frac{\cos x - \sin x}{\sin x + \cos x} dx$

40. $\int \frac{\cot x}{\log \sin x} dx$

41. $\int \frac{\sec^2 x}{a + b\tan x} dx$

42. $\int \frac{\sin x}{1+\cos x} dx$

43. $\int \cos^2 x\, dx$

44. $\int \sin^3 x\, dx$

45. $\int \sin 5x \sin 3x\, dx$

46. $\int \sin^2 x \cos^3 x\, dx$

47. $\int \sin^4 x\, dx$

48. $\int \frac{1}{1+\sin x} dx$

49. $\int \tan^3 x\, dx$

50. $\int \frac{\cos x - \sin x}{1+\sin 2x} dx$

51. $\int \frac{\operatorname{cosec}^2 x}{1+\cot x} dx$

52. $\int \frac{1+x+\cos 2x}{x^2+\sin 2x+2x} dx$

53. $\int \frac{\sec\theta \operatorname{cosec}\theta\, d\theta}{\log \tan\theta}$

54. $\int \frac{\cot\theta\, d\theta}{\log \sin\theta}$

55. $\int \frac{dx}{1+4x^2}$

56. $\int \frac{1-\tan\theta}{1+\tan\theta} d\theta$

57. $\int \frac{1}{x^2} e^{-\frac{1}{x}} dx$

58. $\int \frac{\sin x \cos x\, dx}{a^2\sin^2 x + b^2\cos^2 x}$

59. $\int \frac{dx}{\sin x + \cos x}$

60. $\int e^x\left(\cos^{-1} x - \frac{1}{\sqrt{1-x^2}}\right) dx$

61. $\int e^x\left(\frac{\sin x + \cos x}{\cos^2 x}\right) dx$

62. $\int \tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}\, dx$

63. $\int \cos\left[2\cot^{-1}\left(\sqrt{\frac{1-x}{1+x}}\right)\right] dx$

64. $\int \frac{\sin^{-1} x}{(1-x^2)^{\frac{3}{2}}} dx$

65. $$\int \sqrt{x}\log x\,dx$$

66. $$\int e^x(1+x)\log(xe^x)\,dx$$

67. $$\int \frac{\log x}{(1+x)^2}\,dx$$

68. $$\int e^x \sin^2 x\,dx$$

69. $$\int \cos(\log x)\,dx$$

70. $$\int \log(x+1)\,dx$$

71. $$\int \frac{x^2+1}{(x-1)^2(x+3)}\,dx$$

72. $$\int \frac{\sin\theta\cos\theta}{\cos^2\theta-\cos\theta-2}\,d\theta$$

73. $$\int \frac{dx}{x(x^5+1)}$$

74. $$\int \frac{x^2+1}{(x^2+2)(2x^2+1)}\,dx$$

75. $$\int \frac{\log x}{x(1+\log x)(2+\log x)}\,dx$$

76. $$\int \frac{dx}{1-e^x}$$

ଟିପ୍ପଣୀ

ଉତ୍ତର ମାଳା

26.1

1. $\frac{2}{7}x^{\frac{7}{2}}+1,\ \frac{2}{7}x^{\frac{7}{2}}+2,\ \frac{2}{7}x^{\frac{7}{2}}+3,\ \frac{2}{7}x^{\frac{7}{2}}+4,\ \frac{2}{7}x^{\frac{7}{2}}+5$

2. (a) $\frac{x^6}{6}+c$ (b) $\sin x + c$ (c) 0

3. (a) $\frac{x^7}{7}+c$ (b) $\frac{1}{6x^6}+c$ (c) $\log|x| + c$

   (d) $\frac{\left(\frac{3}{5}\right)^x}{\log\left(\frac{3}{5}\right)}+c$ (e) $\frac{3}{4}x^{\frac{4}{3}}+c$ (f) $\frac{-1}{8x^8}+c$

   (g) $2\sqrt{x}+c$ (h) $9x^{\frac{1}{9}}+c$

4. (a) $-\operatorname{cosec}\theta + c$ (b) $\sec\theta + c$

   (c) $\tan\theta + c$ (d) $-\cot\theta + c$

ଆସ ନିଜେ ନିଜକୁ ପରଖିବା 26.6

1. (a) $\frac{x^2}{2}+\frac{1}{2}x+c$ (b) $-x + \tan^{-1}x + c$

   (c) $x^{10}-\frac{2}{3}x^{\frac{3}{2}}+2\sqrt{x}+c$ (d) $-\frac{1}{x^5}-\frac{3}{4x^4}+\frac{2}{3x^3}+\frac{7}{x}-8x+c$

   (e) $\frac{x^3}{3}-x-\tan^{-1}x+c$ (f) $\frac{x^2}{2}+4x+4\log x+c$

2. (a) $\frac{1}{2}\tan x+c$ (b) $\tan x - x + c$

   (c) $-2\operatorname{cosec} x + c$ (d) $-\frac{1}{2}\cot x+c$

   (e) $-\sec x + c$ (f) $-\cot x + \operatorname{cosec} x + c$

3. (a) $\sqrt{2}\sin x+c$ (b) $-\sqrt{2}\cos x+c$

   (c) $-\frac{1}{2}\cot x+c$

4. (a) $\frac{2}{3}(x+2)^{\frac{3}{2}}+c$ (b) $\frac{-17}{16}\frac{1}{(x+1)^{16}}+c$

ଆସ ନିଜେ ନିଜକୁ ପରଖିବା 26.3

1. (a) $\frac{1}{5}\cos(4-5x)+c$ (b) $\frac{1}{3}\tan(2+3x)+c$

(c) $\log\left|\sec\left(x+\frac{\pi}{4}\right)+\tan\left(x+\frac{\pi}{4}\right)\right|+c$

(d) $\frac{1}{4}\sin(4x+5)+c$

(e) $\frac{1}{3}\sec(3x+5)+c$ (f) $-\frac{1}{5}\operatorname{cosec}(3+5x)+c$

2. (a) $\frac{1}{12(3-4x)^3}+c$ (b) $\frac{1}{5}(x+1)^5+c$

(c) $-\frac{1}{77}(4-7x)^{11}+c$ (d) $\frac{1}{16}(4x-5)^4+c$

(e) $\frac{1}{3}\log|3x-5|+c$ (f) $-\frac{2}{9}\sqrt{5-9x}+c$

(g) $\frac{1}{6}(2x+1)^3+c$ (h) $\log|x+1|+c$

3. (a) $\frac{1}{2}e^{2x+1}+c$ (b) $-\frac{1}{8}e^{3-8x}+c$ (c) $-\frac{1}{4e^{(7+4x)}}+c$

4. (a) $\frac{1}{2}\left(x+\frac{\sin 2x}{2}\right)+c$ (b) $\frac{1}{32}\left(-\frac{3}{2}\cos 2x+\frac{1}{6}\cos 6x\right)+c$

(c) $\frac{1}{2}\left[-\frac{\cos 7x}{7}-\cos x\right]+c$ (d) $\frac{1}{2}\left[\frac{\sin 6x}{6}+\frac{\sin 2x}{2}\right]+c$

ଆସ ନିଜେ ନିଜକୁ ପରଖିବା 26.4

1. (a) $\frac{1}{6}\log|3x^2-2|+c$ (b) $\log|x^2+x+1|+c$

(c) $\log|x^2+9x+30|+c$ (d) $\frac{1}{3}\log|x^3+3x+3|+c$

(e) $\log|x^2+x-5|+c$ (f) $2\log\left|5+\sqrt{x}\right|+c$

(g) $\log|8+\log x|+c$

2. (a) $\frac{1}{b}\log|a+be^x|+c$ (b) $\tan^{-1}(e^x)+c$

ଆସ ନିଜେ ନିଜକୁ ପରଖିବା 26.5

1. (a) $x+\frac{3}{2}\log\left|\frac{x-3}{x+3}\right|+c$ (b) $\tan^{-1}(e^x)+c$

(c) $\frac{1}{2}\tan^{-1}(x^2)+c$ (d) $\frac{1}{3}\sin^{-1}\left(\frac{3x}{4}\right)+c$

(e) $\frac{1}{2}\tan^{-1}(2\tan x)+c$ (f) $\sin^{-1}\left(\frac{x+1}{2}\right)+c$

(g) $\frac{1}{3\sqrt{6}}\tan^{-1}\left(\frac{x+1}{\sqrt{6}}\right)+c$ (h) $\sin^{-1}\left(\frac{x+2}{3}\right)+c$

(i) $\frac{1}{2\sqrt{3}}\sec^{-1}\frac{x}{2}+c$ (j) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan^2\theta-1}{\sqrt{2}\tan\theta}\right)+c$

(k) $\log\left|e^x+\sqrt{1+e^{2x}}\right|+c$ (l) $\sin^{-1}x-\sqrt{1-x^2}+c$

(m) $\sin^{-1}\left(\frac{x-a}{a}\right)+c$ (n) $\frac{1}{4}\sin^{-1}\left(\frac{4}{3}x^3\right)+c$

(o) $\sqrt{x^2+1}+\log\left|x+\sqrt{x^2+1}\right|+c$ (p) $\frac{1}{2}\log\left|\frac{2x+\sqrt{9+4x^2}}{2}\right|+c$

(q) $-\frac{1}{2}\log\left|2\cos\theta+\sqrt{4\cos^2\theta-1}\right|+c$ (r) $\log\left|\tan x+\sqrt{\tan^2 x-4}\right|+c$

(s) $\tan^{-1}\left(\frac{x+2}{1}\right)+c$ (t) $\frac{1}{4}\log\left|x+\sqrt{x^2+\left(\frac{5}{4}\right)^2}\right|+c$

ଆସ ନିଜେ ନିଜକୁ ପରଖିବା 26.6

1. (a) $-x\cos x+\sin x+c$

(b) $\frac{1}{2}(1+x^2)\sin 2x+\frac{x\cos 2x}{2}-\frac{\sin 2x}{4}+c$

(c) $\frac{-x\cos 2x}{2}+\frac{1}{2}\,\frac{\sin 2x}{2}+c$

2. (a) $x\tan x-\log|\sec x|-x+c$

(b) $\frac{1}{6}x^3-\frac{1}{4}x^2\sin 2x-\frac{1}{4}x\cos 2x+\frac{1}{8}\sin 2x+c$

3. (a) $\frac{x^4 \log 2x}{4} - \frac{x^4}{16} + c$ (b) $\left(x - \frac{x^3}{3}\right)\log x - x + \frac{x^3}{9} + c$

(c) $x(\log x)^2 - 2x \log x + 2x + c$

4. (a) $\frac{x^{1-n}}{1-n}\log x - \frac{x^{1-n}}{(1-n)^2} + c$ (b) $\log x[\log(\log x) - 1] + c$

5. (a) $e^{3x}\left[\frac{x^2}{3} - \frac{2x}{9} + \frac{2}{27}\right] + c$ (b) $x\frac{e^{4x}}{4} - \frac{e^{4x}}{16} + c$

6. $\frac{x^2}{2}\left[(\log x)^2 - \log x + \frac{1}{2}\right] + c$

7. (a) $x\sec^{-1}x - \log\left|x + \sqrt{x^2 - 1}\right| + c$

(b) $\frac{x^2}{2}\cot^{-1}x + \frac{x}{2} + \frac{1}{2}\cot^{-1}x + c$

ଆସ ନିଜେ ନିଜକୁ ପରଖିବା 26.7

1. (a) $e^x \sec x + c$ (b) $e^x \log|\sec x + \tan x| + c$

2. (a) $\frac{1}{x}e^x + c$ (b) $e^x \sin^{-1}x + c$

3. $\frac{e^x}{(1+x)^2} + c$ 4. $\frac{e^x}{1+x} + c$

5. $x\tan\frac{x}{2} + c$

6. $\frac{1}{5}e^x(\sin 2x - 2\cos 2x) + c$

ଆସ ନିଜେ ନିଜକୁ ପରଖିବା 26.8

1. (a) $x\sqrt{x^2 - \frac{5}{4}} - \frac{5}{4}\log\left|x + \sqrt{x^2 - \frac{5}{4}}\right| + c$

(b) $\frac{(2x+3)}{4}\sqrt{x^2 + 3x} - \frac{9}{8}\log\left|\left(x + \frac{3}{2}\right) + \sqrt{x^2 + 3x}\right| + c$

(c) $\frac{1}{4}(2x+1)\sqrt{3 - 2x - 2x^2} + \frac{7}{4\sqrt{2}}\sin^{-1}\left(\frac{2x+1}{\sqrt{7}}\right) + c$

2. (a) $4 \log |x-3| - 3 \log |x-2| + c$

(b) $\frac{1}{2}\log|x-4| + \log|x+4| + c$

3. (a) $\frac{x^2}{2} - 2[\log|x-2| + \log|x+2|] + c$

(b) $\frac{11}{9}\log|x-1| + \frac{7}{9}\log(x+2) - \frac{4}{3(x-1)} + c$

4. $\log|x-1| - \frac{3}{(x-1)} - \frac{3}{2(x-1)^2} + c$

5. (a) $\frac{1}{8}\log|1-\sin x| - \frac{1}{8}|1+\sin x|$

$-\frac{1}{4\sqrt{2}}\log\left|1-\sqrt{2}\sin x\right| + \frac{1}{4\sqrt{2}}\log\left|1+\sqrt{2}\sin x\right| + c$

(b) $\log|\sec x + \tan x| - 2\tan\frac{x}{2} + c$

ପାଠଶେଷ / ଅଭ୍ୟାସ କାର୍ଯ୍ୟ

1. $\sec x - \operatorname{cosec} x + c$
2. $\sin x - \cos x + c$
3. $-\cot x - \tan x + c$
4. $\tan x - \cot x - 4x + c$
5. $4\tan^{-1}x + \sin^{-1}x + c$
6. $\tan x - x + c$
7. $-\cot x - x + c$
8. $x - \cos x + c$
9. $x + \cos x + c$
10. $\frac{\sin(7x-\pi)}{7} + c$
11. $\frac{-\cos(3x+4)}{3} + c$
12. $\frac{\tan(2x+b)}{2} + c$
13. $\frac{1}{\sqrt{2}}\log\left|\operatorname{cosec}\left(x-\frac{\pi}{4}\right) - \cot\left(x-\frac{\pi}{4}\right)\right| + c$
14. $\log|\tan^{-1}x| + c$
15. $\log\left|\log\tan\frac{x}{2}\right| + c$
16. $\frac{1}{4}\log|3+4\log\sin x| + c$
17. $\frac{1}{2}\log|\log\tan x| + c$
18. $2\log\left|e^{\frac{x}{2}} - e^{\frac{-x}{2}}\right| + c$
19. $\frac{1}{4}\sec^4 x + c$

20. $-\cos e^x + c$

21. $\dfrac{\sqrt{2x^2+3}}{2}+c$

22. $2\sqrt{\tan x}+c$

23. $\dfrac{1}{6}x\sqrt{(25-9x^2)}+\dfrac{25}{6}\sin^{-1}\left(\dfrac{3}{5}x\right)+c$

24. $\dfrac{1}{2}(x-a)\sqrt{2ax-x^2}+\dfrac{1}{2}a^2\sin^{-1}\left(\dfrac{x-a}{a}\right)+c$

25. $\dfrac{x\sqrt{3x^2+4}}{2}+\dfrac{2}{\sqrt{3}}\log\left|\dfrac{\sqrt{3x}+\sqrt{x^2+4}}{2}\right|+c$

26. $\dfrac{x\sqrt{9x^2+1}}{2}+\dfrac{1}{6}\log\left|3x+\sqrt{1+9x^2}\right|+c$

27. $\left[\dfrac{1}{2}x\sqrt{x^2-a^2}+\dfrac{1}{2}a^2\log\left|x+\sqrt{x^2-a^2}\right|\right]+c$

28. $\dfrac{1}{2}\tan^{-1}\left(\dfrac{\tan x}{2}\right)+c$

29. $\dfrac{2}{\sqrt{3}}\tan^{-1}\left[\dfrac{\tan\left(\dfrac{x}{2}\right)}{\sqrt{3}}\right]+c$

30. $\dfrac{1}{2}\tan^{-1}\left(\dfrac{x-3}{2}\right)+c$

31. $\dfrac{1}{2}\tan^{-1}(2\tan x)+c$

32. $x+\dfrac{a}{2}\log\left|\dfrac{x-a}{x+a}\right|+C$

33. $\dfrac{1}{12}\log\left|\dfrac{\sqrt{9+x^4}-3}{\sqrt{9+x^4}+3}\right|+c$

34. $\dfrac{1}{2\sqrt{3}}\log\left|\dfrac{\sqrt{3}+\tan x}{\sqrt{3}-\tan x}\right|+c$

35. $\dfrac{1}{2\sqrt{2}}\log\left|\dfrac{\tan x-\sqrt{2}}{\tan x+\sqrt{2}}\right|+c$

36. $\dfrac{1}{a}\tan(ax+b)+c$

37. $\log|(2+\log x)|+c$

38. $\dfrac{1}{6}\log(1+x^6)+c$

39. $\log|\sin x+\cos x|+c$

40. $\log|\log(\sin x)|+C$

41. $\dfrac{1}{b}\log|a+b\tan x|+c$

42. $-\log|1+\cos x|+c$

43. $\dfrac{1}{2}\,\dfrac{\sin 2x}{2}+\dfrac{1}{2}x+c$

44. $-\cos x+\dfrac{\cos^3 x}{3}+c$

45. $\dfrac{1}{2}\dfrac{\sin 2x}{2}-\dfrac{1}{2}\dfrac{\sin 8x}{8}+c$

46. $\dfrac{1}{3}\sin^3 x-\dfrac{\sin^5 x}{5}+c$

47. $\dfrac{1}{32}[12x-8\sin 2x+\sin 4x]+c$

48. $\tan x - \sec x + c$

49. $\dfrac{\tan^2 x}{2}+\log|\cos x|+c$

50. $\dfrac{-1}{\cos x+\sin x}+c$

51. $\log\left|\dfrac{1}{1+\cot x}\right|+c$

52. $\dfrac{1}{2}\log|x^2+\sin 2x+2x|+c$

53. $\log|\tan\theta|+c$

54. $\log|\log\sin\theta|+c$

55. $\dfrac{1}{2}\tan^{-1}2x$

56. $\log|\cos\theta+\sin\theta|+c$

57. $e^{-\frac{1}{x}}+c$

58. $\dfrac{1}{2(a^2-b^2)}\log|a^2\sin^2 x+b^2\cos^2 x|+c$

59. $\dfrac{1}{\sqrt{2}}\log\left|\sec\left(x-\dfrac{\pi}{4}\right)+\tan\left(x-\dfrac{\pi}{4}\right)\right|+c$

60. $e^x\cos^{-1}x+c$

61. $e^x\sec x+c$

62. $\dfrac{1}{4}x^2+c$

63. $-\dfrac{1}{2}x^2+c$

64. $\dfrac{x\sin^{-1}x}{\sqrt{1-x^2}}+\dfrac{1}{2}\log|1-x^2|+c$

65. $\dfrac{2}{3}x^{\frac{3}{2}}\left(\log x-\dfrac{2}{3}\right)+c$

66. $xe^x[\log(xe^x)-1]+c$

67. $-\dfrac{1}{1+x}\log|x|+\log|x|-\log|x+1|+c$

68. $\dfrac{1}{2}e^x-\dfrac{e^x}{10}(2\sin 2x+\cos 2x)+c$

69. $\dfrac{x}{2}[\cos(\log x)+\sin(\log x)]+c$

70. $x\log|x+1|-x+\log|x+1|+c$

71. $\dfrac{3}{8}\log|x-1|-\dfrac{1}{2(x-1)}+\dfrac{5}{8}\log|x+3|+c$

72. $-\frac{2}{3}\log|\cos\theta - 2| - \frac{1}{3}\log|\cos\theta + 1| + C$

73. $\frac{1}{5}\log\left|\frac{x^5}{x^5+1}\right| + C$

74. $\frac{1}{3\sqrt{2}}\left[\tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + \tan^{-1}\left(\sqrt{2}x\right)\right] + C$

75. $\log\left|\frac{(2+\log x)^2}{1+\log x}\right| + C$

76. $\log\left|\frac{e^x}{1-e^x}\right| + C$